The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, AUTUMN SEMESTER 2015-2016

MECHANICS OF SOLIDS 2

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR questions

Only silent, self contained calculators with a Single-Line Display or Dual-Line Display are permitted in the examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

In this examination candidates are required to answer FOUR out of SIX questions. If a candidate answers more than the required number of questions, all questions will be marked and the highest marks will be used in the final examination mark.

ADDITIONAL MATERIAL: Graph Paper

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

1. A thin-walled cylinder is subjected simultaneously to an internal pressure of 100 kPa, an axial compressive load of 5 kN, and a torsional moment of 10 kNm. The radius of the cylinder is 200 mm, and the wall thickness is 1 mm.

   (a) Determine the state of stress on a plane stress element located on the surface of the cylinder, include a sketch. [12]

   (b) Determine the magnitude of the in-plane principal stresses and maximum shear stress, include a sketch of Mohr's Circle on the graph paper provided. [13]

2. A rigid mechanical component as shown in Figure Q2 is hinged at point C, held by two identical wires at points A and B, and is subjected to a vertical load *W* at point D. Each wire has an axial rigidity, *EA*, of 200 kN and a thermal expansion coefficient of 12 x 10^{-6}/°C.

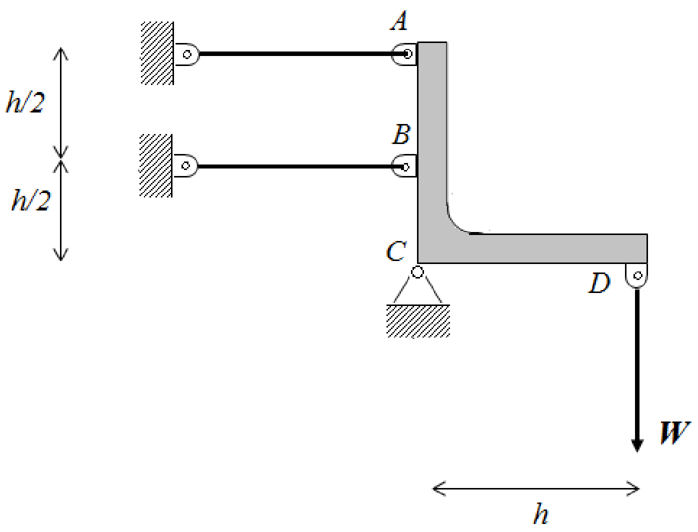

Figure Q2

   (a) If the applied vertical load, *W*, is 1 kN, calculate the tensile forces in the two wires. [5]

   (b) While the load *W* is being applied, the temperature of the strings is raised by 100°C, calculate the tensile force in the wires. [10]

   (c) Determine how many degrees the temperature has to be increased so that only the wire at point B becomes slack (i.e. the force in this wire is zero). [10]

3. A rectangular section beam (width 16 mm, height 18 mm) is made from an elastic-perfectly-plastic material with a yield stress of 210 MPa and a Young's modulus of 195 GPa. It is loaded by a bending moment of 205 Nm that causes the beam to sag.

   (a) Determine the stress state in the beam when the moment is applied. [15]

   (b) Determine the residual curvature in the beam that is present once the bending moment is removed. [10]

4. (a) With the use of suitable diagrams explain the development of a fatigue crack, from initiation in an un-cracked component with smooth surfaces to complete failure. [10]

   (b) Draw a stress versus time loading curve that could be used in a fatigue test. Label the stress amplitude, mean stress and stress range. [5]

   (c) A component is made from a material with an ultimate tensile strength of 294 MPa. The component will be cyclically loaded such that the mean stress is 45 MPa. Using a modified Goodman line, determine the maximum allowable stress amplitude if a safety factor of 1.25 is to be achieved. Assume an endurance limit for the material of 105 MPa and a fatigue notch strength reduction factor of 2 for the component. [10]

5. (a) Present the stiffness equations for a 1D bar element in matrix form. [5]

The pin-jointed framework ABC is subjected to an external load as shown in Figure Q5. If each member has a length, L, of 1 m and a value of the product AE of 200 MN

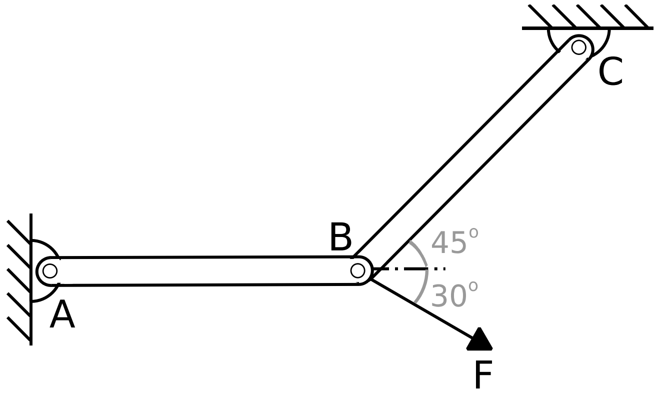

Figure Q5

(b) Construct the stiffness matrix of the structure. [10]

If the applied load, F, is 20 kN:

(c) Determine the horizontal and vertical displacements at point B. [4]

(d) Determine the reaction forces at points A and C. [6]

The stiffness matrix of a truss element is:

$$[k_e] = \left(\frac{AE}{L}\right)\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

where the angle θ is defined as the angle of inclination of the element measured anti-clockwise from the horizontal axis.

6. The Tresca or τ_{max} criterion states that the material will yield if:

$$\sigma_1 - \sigma_3 \geq \sigma_y \text{ for } \sigma_1 > \sigma_2 > \sigma_3$$

The von Mises yield criterion states that the material will yield if:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \geq 2\sigma_y^2$$

(a) Sketch (without using graph paper) the yield boundaries for Tresca and von Mises, for a two-dimensional stress state, on the σ_1 - σ_2 plane ($\sigma_3 = 0$). [3]

(b) Sketch (without using graph paper) the yield surfaces for Tresca and von Mises for a three-dimensional stress state. [3]

(c) Sketch (without using graph paper) the decomposition of the stress into hydrostatic and deviatoric components. [3]

(d) A shaft is to be made from a material with a uniaxial yield stress of 400 MPa. The design loads for the shaft are a torque, *T*, of 8 kNm and a bending moment, *M*, of 4 kNm. Assuming a safety factor of 2, calculate the radius of the shaft based on:

i) The Tresca yield criterion
ii) The von Mises yield criterion

[16]